SUPERSTRINGS FROM THEORIES
WITH $N > 1$ WORLD–SHEET SUPERSYMMETRY

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Abstract String theories with (N, N') local world-sheet supersymmetries are related to each other by marginal deformations. This connects $N = 1$ and $N = 0$ theories in which the target-spaces are interpreted as space-times, $N = 2$ theories in which the target spaces can be interpreted as world-volumes, and theories with $N \geq 3$, in which the central charge vanishes – theories with zero target-space dimensions.

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1 Introduction

The recent progress in understanding non-perturbative features of superstrings has revealed many striking relationships between apparently different theories. Theories with very different fundamental excitations are simply different expansions of the same theory – the solitonic p-branes of one theory, wrapped in a variety of ways around a compact space, may become the fundamental strings of another. Such dualities can relate theories that superficially have strikingly different properties – for example, a theory in a particular space-time dimension can be related to a theory in another dimension. An intriguing aspect of these dualities is that the rôles of the world-volume of a p-brane and the target space are often not distinct – the world-volume of one theory may be the target space of another. One aim of this paper is to indicate how the distinction between the embedding space and the world-volume could emerge from a more fundamental formulation of the theory by building upon an idea in [1]

It seems possible that string theories with local $N = 2$ supersymmetry [2, 3] have a special rôle to play in such a reformulation of string theory. Evidence for this arises from several sources. (a) The $(2,0)$ heterotic version of such theories can have a target space with $[1,1]$ signature which may be interpreted as the *world-sheet* of the bosonic string [4]. [In order to avoid notational confusion the left-moving and right-moving supersymmetries will be denoted by curved brackets $(.,.)$ while the signature will be denoted by square brackets $[.,.]$ in the following.] As shown in [5] the target space may also have signature $[2,1]$ (the ambiguity in the target-space dimension is associated with an ambiguity in the choice of a null projector), in which case it may be interpreted as a bosonic version of the membrane world-volume [6, 7]. (b) The quantization of the heterotic $(2,1)$ theories was shown in [5] to also give theories with either $[1,1]$ or $[2,1]$ target spaces. Those theories with $[1,1]$ target spaces have precisely the field content needed to describe the world-sheets of the critical superstring theories [6] (see also [8]). The different world-sheet theories – I, IIA, IIB, heterotic, as well as the bosonic – correspond simply to different choices of spin structures and orbifolding in various ways. The case of the $(2,1)$ theory with $[2,1]$ target-space signature has the field content needed to describe the world-volume of the membrane of eleven-dimensional M-'theory'. Thus, by making changes in the

boundary conditions of the $(2,1)$ world-sheet theory the target space can be transformed into the world-volumes of any of the theories (the superstrings and M-'theory') that are related by duality symmetries. (c) The $N = 2$ theory in its pure $(2,2)$ form describes self-dual gravity in $[2,2]$ (or $[4,0]$) target-space dimensions while the heterotic theories can be viewed as projected versions of self-dual gravity coupled to self-dual matter (field theories of this type are described in [10] and references therein). A point of view expressed in [5] emphasizes that the $(2,2)$ super-world-sheet really has four bosonic coordinates – a complex parameter z conjugate to the world-sheet hamiltonian and a complex parameter u conjugate to the $U(1)$ charge of the $N = 2$ superconformal group. It can be interpreted as a brane of signature $[2,2]$, which is the same signature as the target space – in fact, there is a symmetry that interchanges the target space and the world-volume (see also [9]).

The idea that the world-sheet of one theory may be the target space of another is attractive although it seems fraught with calculational and conceptual problems. For example, the calculation of a conventional string tree diagram would require second-quantized $N = 2$ string field theory in a target space-time that has the appropriate topology. In any case, this view-point still places emphasis on the distinction between the world-volume and the embedding space. The observations in this paper may be taken as an indication that both world-sheet and target space might be described as different manifestations of the same underlying theory. We will describe marginal deformations that transform superstring theories with different target-space dimensions into each other. Our description will identify marginal operators that are used to deform the target-space moduli. From the point of view of the $(2,1)$ theory a subset of these theories are connected by deformations of the world-sheet moduli. A system in which a mapping between world-sheet and target-space moduli can be identified is given in [11].

These deformations connect theories with target spaces that describe space-time (such as the $(1,1)$ and $(1,0)$ superstrings) to theories with target spaces that describe world-volumes (such as the $(2,1)$ superstring) and to theories which do not have any target space at all (theories with $N \geq 3$).

The initial observations (in section 2) will be based on considering marginal deformations of a $(2,2)$ theory. Such deformations, which correspond to twisting various fields, connect all critical string theories with (N, N') world-sheet supersymmetries (where $0 \leq N, N' \leq 2$). In this manner string theories with target spaces that may be interpreted as world-volumes are

linked to conventional superstring theories by marginal deformations. The initial $(2, 2)$ theory considered in section 2 consists of the usual set of critical fields describing the $[2, 2]$ target space supplemented by a sector consisting of a 'topological package' of matter and ghosts that make cancelling contributions to the central charge. Such a package is defined to consist of four 'BRST quartets', each comprising fermionic fields with conformal spins $(1, 0)$ that will be denoted $(\lambda_1^\alpha, \rho_1^\alpha)$ ($\alpha = 1, 2, 3, 4$) and their compensating bosonic fields (which are also $(1, 0)$ fields) which will be denoted $(\overline{F}_1^\alpha, F_1^\alpha)$. The fermionic fields $(\lambda^\alpha, \rho^\alpha)$ might be related to Green–Schwarz fermions in a manifestly supersymmetric formulation of the theory. In much the same way as for the topological packages considered in [1] these fields have the BRST-exact action,

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\begin{aligned}
I_{top} &= i \int d^2 z \sum_{\alpha=1}^{4} s(\overline{F}_1^\alpha \partial_{\overline{z}} \rho_1^\alpha) \\
&= \int d^2 z \sum_{\alpha=1}^{4} (i\lambda_1^\alpha \partial_{\overline{z}} \rho_1^\alpha - \overline{F}_1^\alpha \partial_{\overline{z}} F_1^\alpha),
\end{aligned}
\tag{1.1}
$$

where the BRST transformations, $s\rho_1^\alpha = iF_1^\alpha$, $sF_1^\alpha = 0$, $s\overline{F}_1^\alpha = \lambda_1^\alpha$ and $s\lambda_1^\alpha = 0$ have been used. Evidently any number of such topological packages can be added to any theory without introducing any anomalies.

The process of twisting the gravitino ghosts in order to reduce the supersymmetry will also be described in section 2. This changes the weight of one of the $(3/2, -1/2)$ pairs to $(1/2, 1/2)$ thereby converting it into 'matter', resulting in a theory with reduced supersymmetry. The anomaly-free condition can be maintained if compensating twists are made on the $(1, 0)$ fermions in a topological package, $(\lambda_1^\alpha, \rho_1^\alpha)$, that transforms them into conformal weight $(1/2, 1/2)$ fields. A version of the $(2, 1)$ heterotic theory is obtained after twisting the right-movers in this manner. A further twist on the left-movers results in $(1, 1)$ theories – the type II superstrings. In order to proceed to theories with lower supersymmetry it is necessary to add a second topological package, $(F_2^\alpha, \overline{F}_2^\alpha); (\lambda_2^\alpha, \rho_2^\alpha)$ in the starting $N = 2$ theory. Then the number of fields in the action is sufficient to allow final deformations giving rise to the $(1, 0)$ heterotic string and the $(0, 0)$ bosonic string.

The presence of the extra topological matter fields suggests that the $(2, 2)$ theory could itself be considered to have descended from a theory with more supersymmetry. Specifically, it will be shown in section 3 that there are marginal deformations of a critical theory with the 'large' $N = 4$ local world-sheet supersymmetry [3, 12] that correspond to various 'twistings' that take the theory to all possible critical (N, N') theories where $1 \leq N, N' \leq 4$. The critical dimension

of this theory is zero – in other words the total central charge of the ghosts for this large local symmetry is $c_{ghosts}^{(4)} = 0$. This means that the simplest possible $(4,4)$ theory is one with no matter at all – matter can only be added in the form of topological packages with $c_{matt}^{(4)} = 0$. In the first instance we will consider the purely gravitational $N = 4$ theory in which there are no such extra packages. The $N = 4$ superalgebra consists of the Virasoro generators together with seven generators of an affine $SU(2) \times SU(2) \times U(1)$ and eight fermionic generators. The $(4,4)$ world-sheet theory therefore has eight bosonic generators for both left-movers and right-movers. Just as the $(2,2)$ string super world-sheet can be thought of as a $[2,2]$ world-volume when the Kac–Moody parameters are included as bosonic dimensions, the $(4,4)$ world-sheet can be interpreted as an $[8,8]$ dimensional world-volume. This is suggestive of an underlying connection with octonions [7].

Again the supersymmetry can be reduced by using compensating twists on a gravitino ghost and other ghost fields, resulting in the $N = 3$ theory (although this theory can also be obtained simply by integrating out an anomaly free set of fields, reflecting the fact that the $N = 3$ theory also has $c_{ghosts}^{(3)} = 0$). After two steps for both the right-movers and the left-movers the $(2,2)$ theory is obtained, including the first set of topological matter fields considered in section 2. This leads, as before, to the string theories with $N, N' = 1, 2$. To recover the theories with at least one $N = 0$ chiral sector requires the $N = 4$ theory to contain multiplets of matter fields in addition to the gravitational sector – these necessarily arise as $c = 0$ topological packages.

Another reason for including additional topological packages arises from considering chiral deformations of the $(4,4)$ theory to give heterotic $(4, N)$ $(N \leq 2)$ theories. There are strong constraints imposed by requiring that the $N = 4$ fermionic ghosts have both left-moving and right-moving components. These can only be satisfied if extra right-moving topological packages of fields are added to the initial theory, as will be discussed in section 3.3.

Section 4 presents speculations as to how the incorporation of these extra topological packages may arise naturally in theories with $N \geq 5$ local world-sheet supersymmetry. Such theories have vanishing central charge as in the case of $N = 4$, but they have a bigger ghost spectrum, determined from the antisymmetric representations of $O(N)$ [13]. After suitable deformations the extra ghosts give rise to sufficiently many 'matter' fields to extend the arguments of the earlier sections to include string theories with $N = 0$ sectors.

The variety of target spaces obtained by deforming the same initial underlying string theory

(with $N \geq 4$) include those that can also be interpreted as world-sheets of the underlying string theory itself. We do not attempt to answer the question of whether the $[2, 1]$ target space of the heterotic $(2, 1)$ string – the suggested effective M-brane – can also be interpreted as a deformation of the underlying world-volume theory to a three-dimensional theory. Such a relation between theories in two and three dimensions seems possible by analogy with the equivalence between the three-dimensional level-k Chern–Simons theory in the infinite k limit and the two-dimensional $\phi * F$ system based on the same gauge group [14, 15]. Deformations of either of these equivalent formulations relate theories with different dimensions.

Other somewhat different arguments have previously been advanced for considering the embeddings of superstrings with lower supersymmetry within those with higher supersymmetry [16, 17, 18].

2   N=2 world-sheet supersymmetry

The basic idea of the construction is inspired by [1] where the type II and heterotic strings were obtained as different decompositions of the same action. This action, which was shown to be a twisted version of a topological sigma model with bosonic and fermionic coordinates will be generalized to a theory with local $N = 2$ supersymmetry. However, since we are now expecting to relate theories with different target-space dimensions it will be necessary to twist some of the ghost fields so that they become extra matter fields.

Let us review the various conformal components of the 'standard' critical $(2, 2)$ supersymmetric superstring. The fields of extended $N = 2$ supergravity can be described by the holomorphic Beltrami differential μ, two gravitini α_1, α_2, and a $U(1)$ gauge field A, together with their antiholomorphic counterparts. The associated ghost sector consists of the following fields. The conformal weight-$(2, -1)$ fermionic ghosts for general coordinate invariance (b, c) contribute $c = (-26, -26)$ to the central charge.[1] Two bosonic ghosts for the $N = (2, 2)$ supersymmetry, (β^1, γ^1) and (β^2, γ^2), each of conformal-weight $(3/2, -1/2)$, contribute a total of $c = (22, 22)$. The fermionic conformal-weight $(1, 0)$ ghosts for the local $U(1)$ gauge subalgebra of the $N = 2$ superconformal symmetry, (f, g), contribute $c = (-2, -2)$. Altogether the ghost sector contributes $c_{ghosts}^{(2)} = (-6, -6)$ to the central charge. The compensating matter sector includes

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[1]The left-moving and right-moving central charges will be denoted $c = (c_L, c_R)$.

four bosonic dimensions, $P^\mu = \partial X^\mu$, contributing $c = (4,4)$, and four fermionic spin-1/2 fields, ψ^μ, contributing $c^{(2)}_{matt} = (2,2)$. The target space of this theory is therefore interpreted as four-dimensional. However, the $N = 2$ supersymmetry restricts this to even signature, $[0,4]$ or $[2,2]$[2] For the present purposes we will only consider the $[2,2]$ case.

This standard critical model can be generalized by adding extra topological matter. The particular topological packages of fields that we will consider consist of the fields $(\overline{F}^\alpha_1, F^\alpha_1); (\lambda^\alpha_1, \rho^\alpha_1)$ and $(\overline{F}^\alpha_2, F^\alpha_2); (\lambda^\alpha_2, \rho^\alpha_2)$. The $F_i, \overline{F}_i$ are bosonic while λ_i, ρ_i are fermionic. These fields all have conformal weight-$(1,0)$ and each bosonic pair contributes $c = (2,2)$ for each value of α while each fermionic pair contributes $c = (-2,-2)$ (and $\alpha = 1, \cdots, 4$). Thus, the total central charge from this sector vanishes. The field content of the $(2,2)$ theory is summarized in table 1.

	$N = 2$			$N = 1$			$N = 0$		
	field	weight	c	field	weight	c	field	weight	c
-	(b,c)	$(2,-1)$	-26	(b,c)	$(2,-1)$	-26	(b,c)	$(2,-1)$	-26
+	(β^1, γ^1)	$(\frac{3}{2}, -\frac{1}{2})$	11	(β^1, γ^1)	$(\frac{3}{2}, -\frac{1}{2})$	11	$(\beta^1, \gamma^1)^T$	$(\frac{1}{2}, \frac{1}{2})$	-1
+	(β^2, γ^2)	$(\frac{3}{2}, -\frac{1}{2})$	11	$(\beta^2, \gamma^2)^T$	$(\frac{1}{2}, \frac{1}{2})$	-1	$(\beta^2, \gamma^2)^T$	$(\frac{1}{2}, \frac{1}{2})$	-1
-	(f,g)	$(1,0)$	-2	(f,g)	$(1,0)$	-2	(f,g)	$(1,0)$	-2
+	P^μ	$(1,0)$	4	P^μ	$(1,0)$	4	P^μ	$(1,0)$	4
-	ψ^μ	$(\frac{1}{2}, \frac{1}{2})$	2	ψ^μ	$(\frac{1}{2}, \frac{1}{2})$	2	ψ^μ	$(\frac{1}{2}, \frac{1}{2})$	2
+	$(\overline{F}^\alpha_1, F^\alpha_1)$	$(1,0)$	8	$(\overline{F}^\alpha_1, F^\alpha_1)$	$(1,0)$	8	$(\overline{F}^\alpha_1, F^\alpha_1)$	$(1,0)$	8
-	$(\lambda^\alpha_1, \rho^\alpha_1)$	$(1,0)$	- 8	$(\lambda^\alpha_1, \rho^\alpha_1)^T$	$(\frac{1}{2}, \frac{1}{2})$	4	$(\lambda^\alpha_1, \rho^\alpha_1)^T$	$(\frac{1}{2}, \frac{1}{2})$	4
+	$(\overline{F}^\alpha_2, F^\alpha_2)$	$(1,0)$	8	$(\overline{F}^\alpha_2, F^\alpha_2)$	$(1,0)$	8	$(\overline{F}^\alpha_2, F^\alpha_2)$	$(1,0)$	8
-	$(\lambda^\alpha_2, \rho^\alpha_2)$	$(1,0)$	- 8	$(\lambda^\alpha_2, \rho^\alpha_2)$	$(1,0)$	-8	$(\lambda^\alpha_2, \rho^\alpha_2)^T$	$(\frac{1}{2}, \frac{1}{2})$	4

TABLE 1: The fields of the holomorphic sector of the $N = 2$ theory are listed together with their conformal weights and the total contribution to the central charge, c, of the Virasoro algebra ($\mu = 1, \cdots, 4$ and $\alpha = 1, \cdots, 4$). The deformations that reduce the supersymmetry to $N = 1$ and $N = 0$ are indicated in the last six columns. The first column indicates the fermionic grading of each field which is unchanged by the deformations.

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[2]The target-space coordinates are naturally grouped in complex pairs , $\partial X^i, \partial X^{\bar{i}}, \psi^i, \psi^{\bar{i}}$ with $i, \bar{i} = 1, 2$.

The gauge-fixed lagrangian for this $(2,2)$ model can be written as

$$I_B = I^{N=2} + I_{top},\qquad(2.1)$$

where the second term is the BRST-exact expression (1.1) involving only the fields in the topological package, which simply decouple from the space of ρ-independent observables. The first term in the action is given by,

$$I^{N=2} = \int d^2z[b\partial_{\bar{z}}c + \beta_1\partial_{\bar{z}}\gamma_1 + \beta_2\partial_{\bar{z}}\gamma_2 + f\partial_{\bar{z}}g$$

$$+ \sum_{\mu=1}^{4}\left(\frac{1}{2}\partial_{\bar{z}}X^\mu\partial_z X^\mu - i\psi^\mu\partial_z\psi^\mu\right) + \text{complex conjugate}].\qquad(2.2)$$

This action can also be interpreted (generalizing the expression in [1] to the $N=2$ case) as a topological σ model in the gauge defined by the conditions on the gauge fields,

$$\mu = 0, \qquad \alpha_i = 0, \qquad A_z = 0,\qquad(2.3)$$

(the last of these conditions gives rise to the BRST-exact term $\int d^2zs(fA_{\bar{z}}) = \int f\partial_{\bar{z}}g$ in (2.2)) and the holomorphic conditions on the 'matter' fields,

$$\partial X^1 + i\partial X^4 = 0 = \partial X^2 + i\partial X^3, \qquad \partial\rho_i^\alpha = 0,\qquad(2.4)$$

together with corresponding conditions in the anti-holomorphic sector (strictly speaking these conditions are imposed in the usual gaussian manner).

The topological sector decouples and thus appears to be redundant. However, after applying appropriate marginal deformations which result in twisting some, and eventually all, of these extra fields they will transmute into matter fields of strings with less local supersymmetry. The theories defined by arbitrary deformations may not be physically relevant but the physically interesting theories with $(2,1)$, $(1,1)$, $(1,0)$ and $(0,0)$ local supersymmetry are obtained by specific finite deformations.

2.1 Marginal deformations

We will be interested in deformations which twist various fields with conformal weights λ^i in a way that maintains the vanishing of the total central charge. If these deformations are non-chiral they can be expressed as the sum of linear dilaton terms in the action. The action for a

set of N such fields, Φ^i, is given by

$$\sum_{i=1}^{N} I_{k_i} = \sum_{i=1}^{N} \int d^2 z \frac{1}{2\pi} \sqrt{g} \left(\partial_{\bar{z}} \Phi^i \partial_z \Phi^i - \frac{i}{2} k^i R^2 \Phi^i \right).$$ (2.5)

where $R^{(2)}$ is the world sheet curvature and the parameters k^i define the central charge $c = 2 \sum_{i=1}^{N} \epsilon(6\lambda^{i2} - 6\lambda^i + 1) = \sum_{i=1}^{N} (1 - 3k^{i^2})$ (where $\epsilon = +1$ for bosons and -1 for fermions). Thus, the total central charge is unaltered by the presence of such terms if $\sum_i k^{i^2} = 0$ (i.e., if k^i is a null vector) and the $R^{(2)}$-dependent terms do not then modify the total central charge of the free bosonic system.

The main way in which this formula enters in the following is in the deformations that simultaneously transform a bosonic (β, γ) system with $k_\beta \equiv k^5 = 2$ and the fermionic fields in a topological package, $(\lambda^\alpha, \rho^\alpha)$, which have $k_\lambda^\alpha \equiv k^\alpha = -1$ ($\alpha = 1, \cdots, 4$). After the deformation the fields end up with $k^\alpha = k^5 = 0$. A one-parameter family of deformed theories with vanishing central charge can therefore be defined by choosing continuous deformations $k^5 = -2k^\alpha = 2(1 - m)$ with $0 \le m \le 1$. The Minkowski signature five-vector, (k^5, k^α), is manifestly null at every intermediate value of m. However, only for the special discrete values at the endpoints $m = 0$ and $m = 1$ are these assured to be consistent unitary string theories.

More generally, for chiral fields, the twisting procedure is equivalent to a change of field variables in which the relevant fields are multiplied by powers of a gravitino ghost, γ, as described in [1], [19] in the case of the $N = 1$ theory. Thus, for the example in the previous paragraph the field redefinitions may be of the form,

$$\gamma' = \gamma^{1/(1-2m)}, \qquad \rho' = \gamma^{-n} \rho,$$

$$\beta' = \gamma^{-2m/1-2m}(1 - 2m)\left[\beta + n\gamma^{-1}\lambda\rho \right], \qquad \lambda' = \gamma^n \lambda,$$ (2.6)

which ensure that $\int(\lambda^\alpha \bar{\partial} \rho^\alpha + \beta \bar{\partial} \gamma) = \int(\lambda'^\alpha \bar{\partial} \rho'^\alpha + \beta' \bar{\partial} \gamma')$. A priori the parameters m and n are independent, taking the values $m = 0$, $n = 0$ for the undeformed theory and $m = 1$, $n = 1$ for the final end-point of the marginal deformation. The BRST transformations of the fields at intermediate values of m and n follow simply from those of the original fields. It is easy to see that the condition for having a one-parameter family of anomaly-free theories that interpolates between the end-point theories requires the choice,

$$n = -\frac{m}{1 - 2m},$$ (2.7)

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so that $\gamma' = \gamma^{1-2n}$. For generic values of m these theories are probably ill-defined although there may be well-defined theories at rational values of this parameter. The BRST charge Q can be defined by this twisting procedure at any point along the path of deformations starting from the initial theory (although the explicit expression is rather lengthy and is omitted here).

2.2 Marginal deformations of the $N = 2$ theory.

This procedure is now applied to the $N = 2$ theory shown in the left-hand column of table 1. The first step consists of simultaneously twisting the right-moving (β^2, γ^2) and $(\lambda_1^\alpha, \rho_1^\alpha)$ fields. These fields are transformed into $(\beta^2, \gamma^2)^T$ and $(\lambda_1^\alpha, \rho_1^\alpha)^T$ of conformal weight $(1/2, 1/2)$. The left-moving sector remains as the untransformed model with $N = 2$ supersymmetry. After the transformation the field content in the right-moving sector (shown in table 1) consists of the familiar N=1 ghosts – the (b, c) and (β^1, γ^1) fields – as well as the fermionic (f, g) and bosonic $(\beta^2, \gamma^2)^T$ fields. Although these extra ghosts are not needed in the usual $(1, 1)$ theory they arise here because of the pairing of left-moving and right-moving scalar fields in the $(2, 1)$ theory. Thus, the $(\overline{f}, \overline{g})$ ghosts for the $U(1)$ symmetry are paired with the right-moving fields, (f, g). Right-moving $N = 1$ local supersymmetry requires, in addition, the presence of the bosonic right-moving fields $(\beta^2, \gamma^2)^T$. The deformed theory thus possesses the appropriate $(2, 1)$ supersymmetry ghost system (as in [5])

The matter fields of this $(2, 1)$ model include those of the original $(2, 2)$ theory together with $(\overline{F}_1^\alpha, F_1^\alpha)$ which may now be interpreted as eight real bosonic fields adding $[0, 8]$ right-moving target-space dimensions to the $[2, 2]$ already present. Consistency requires these to be compactified on a euclidean self-dual lattice, namely, the E_8 lattice. These bosons can also be written in terms of 16 $(\frac{1}{2}, \frac{1}{2})$ fermions which can be grouped with the eight right-moving fermions, $(\lambda_1^\alpha, \rho_1^\alpha)$, to give more general target-space theories by appropriate choices of spin structures (or GSO projections) [6]. These are right-moving bosonic fields of the usual $(2, 1)$ heterotic case.

The four fermionic matter fields of the $N = 2$ theory, ψ^μ, are similarly supplemented by $(\lambda_1^\alpha, \rho_1^\alpha)^T$, giving a total of twelve fermions which are superpartners of the bosonic matter fields. The target-space theory is therefore described as a heterotic theory with $[2, 2]$ signature for the left-movers and $[10, 2]$ for the right-movers. As in [5, 6] the gauge constraint on the $U(1)$ current leads to a null reduction that eliminates one of the time directions and results in a target-space

that is either $[1, 1]$ or $[2, 1]$. This null reduction pairs the (f, g) ghosts in a topological package with one of the time-like P^μ fields and either a space-like P^μ or The topological package consisting of $(\overline{F}_2^\alpha, F_2^\alpha); (\lambda_2^\alpha, \rho_2^\alpha)$ descends untouched to the $N = (2, 1)$ theory.

It is now obvious that a similar marginal deformation of the left-moving sector will change the theory with $(2, 1)$ supersymmetry to one with $(1, 1)$ supersymmetry. This defines the usual type II theories in an enlarged $[10, 2]$ target space together with the ghosts (f, g) and $(\beta^2, \gamma^2)^T$. These ghosts are associated with gauge constraints that can be used to reduce to the usual critical $[9, 1]$ theory by forming two BRST topological quartets of fields – the first consisting of (f, g) with two of the spin-$(1, 0)$ bosonic fields and the second of $(\beta^2, \gamma^2)^T$ with two of the spin-$(1/2, 1/2)$ fermionic fields. The $(\overline{F}_2^\alpha, F_2^\alpha); (\lambda_2^\alpha, \rho_2^\alpha)$ topological package is again untouched and is simply appended to the $N = (1, 1)$ model.

Strictly speaking, if the twisting is carried out independently for the left-movers and the right-movers the matter fields describe compactified target spaces. More generally, BRST-exact terms can be added to the action that relate the two world-sheet directions, leading to extra zero modes associated with uncompactified dimensions. For example, denoting the left-moving and right-moving bosonic fields $(F_1, \overline{F}_1)$ by $(F_L, \overline{F}_L)$ and $(F_R, \overline{F}_R)$, gives a first-order action,

$$\int d^2 z \left(F_L \partial \overline{F}_L + F_R \overline{\partial F}_R + F_L F_R \right), \tag{2.8}$$

where the last term (which has ghost number 2) is of the form $\int d^2 z s(F_L \rho_R)$. Integrating out F_L and F_R leaves the second-order action, $\int d^2 z \overline{F}_L \partial \overline{\partial F}_R$. If F_L^α and F_R^α are real their zero modes describe extra dimensions of signature $[4, 4]$. If they are to describe purely spatial dimensions it is necessary to continue them to complex values and impose the reality condition, $\overline{F}_R^* = \overline{F}_L$.

The presence of extra topological sectors allows further twistings that reduce the $(1, 1)$ theory to $(1, 0)$. This is obtained by the marginal deformation that twists (β^1, γ^1) into $(\beta^1, \gamma^1)^T$ and simultaneously twists the fermionic components of the topological package from (λ_2, ρ_2) into $(\lambda_2, \rho_2)^T$. The right-moving ghosts now comprise (b, c) together with two BRST quartets consisting of the bosonic fields $(\beta^1, \gamma^1)^T$ and $(\beta^2, \gamma^2)^T$ together with two of the fermionic fields of the same conformal weight, $(\lambda_1^4, \rho_1^4)^T$, $(\lambda_2^4, \rho_2^4)^T$. This leaves twelve right-moving Weyl fields, $(\lambda_1^i, \rho_1^i)^T$ and $(\lambda_2^i, \rho_2^i)^T$ (where $i = 1, 2, 3$). In addition there are eight right-moving chiral bosons, $(\overline{F}_2^\alpha, F_2^\alpha)$ as well as the four original right-moving fermions, ψ^μ, in the matter sector. This gives a total of eight right-moving bosons and sixteen fermions, which corresponds to a

version of the usual content of the right-moving sector of the heterotic string[3].

Finally, the conformal spin of the left-moving gravitino ghosts, $(\overline{\beta}^1, \overline{\gamma}^1)$, may be deformed together with the second topological package of left-moving fields to give the $(0,0)$ bosonic string theory. After bosonising the sixteen left-moving and right-moving fermions they can be identified with eight toroidally compactified bosons. Together with the bosonic fields $(\overline{F}_1^\alpha, F_1^\alpha)$, $(\overline{F}_2^\alpha, F_2^\alpha)$ and their antiholomorphic partners, these make up an extra sixteen dimensions of the 26-dimensional target space of bosonic string theory.

3 Deformations of the large $N = 4$ theory.

The presence of the extra topological packages in the $(2,2)$ theory suggests that it might be embedded naturally in a theory with higher supersymmetry. We will therefore consider the purely gravitational theory with the 'large' $N = 4$ local superconformal symmetry [3].

The generators of the large $N = 4$ superconformal algebra [12] consist of the holomorphic and anti-holomorphic components of the energy-momentum tensor, the four fermionic super-currents, the seven generators of a $U(1) \times O(4)$ (or $U(1) \times SU(2) \times SU(2)$) affine algebra and four more fermionic 'internal symmetry' generators. Each of these generators can be associated with a two-dimensional gauge field. The purely gravitational action for this $N = 4$ theory may then be deduced by setting these gauge fields to zero by a gauge choice that is implemented in a BRST invariant manner (again generalizing the discussion in the $N = 1$ case).

The ghosts for the various gauge symmetries will be denoted by c (for reparameterizations), γ^a (for the four local supersymmetries with $a = 1, 2, 3, 4$), c^{+i}, c^{-i} (for the $SU(2) \times SU(2)$ gauge symmetry, where $i = 1, 2, 3$), ϵ^a (for the four additional fermionic internal symmetries) and g (for the local $U(1)$ symmetry). Ghost fields of integer weight are fermions while the rest are bosons. Correspondingly, in the generalized conformal gauge (in which the gauge fields vanish) the antighosts are b, β^a, $b^{\pm i}$, δ^a and f which are respectively associated with c, γ^a, $c^{\pm i}$, ϵ^a and g.

The ghost action with the large $N = 4$ worldsheet invariance is given by

$$I^{N=4} = \int d^2z \left(b\partial_{\overline{z}}c + \sum_{a=1}^{4} (\beta^a \partial_{\overline{z}} \gamma^a + \delta^a \partial_{\overline{z}} \epsilon^a) \right.$$

---

[3]This content is conventionally expressed in terms of 16 chiral bosons or 32 chiral fermions.

$$+ \sum_{i=1}^{3} \left(b^{+i} \partial_{\bar{z}} c^{+i} + b^{-i} \partial_{\bar{z}} c^{-i} \right) + f \partial_{\bar{z}} g \right). \tag{3.1}$$

This can be expressed as the s-exact action

$$I^{N=4} = \int d^2 z \sum_{\text{allsectors}} s \left(B_{(\mathcal{A})} \mathcal{A} \right), \tag{3.2}$$

where $\mathcal{A}$ stands for the $\bar{z}$ components of the gauge fields with corresponding antighosts $B_{(A)}$. This action enforces the gauge conditions $A = 0$ in a BRST invariant manner.

This ghost system is anomaly free. The central charges of the pairs (b, c), (β^a, γ^a), (ϵ^a, δ^a), (b^{+i}, c^{+i}), (b^{-i}, c^{-i}) and (f, g) give contributions to the central charge of the Virasoro algebra equal to -26, $4 \times 11 = 44$, $4 \times (-1) = -4$, $3 \times (-2) = -6$, $3 \times (-2) = -6$ and -2, respectively. The total central charge, $c_{ghost}^{(4)}$, therefore vanishes as commented on earlier. This means that the only other fields that can be added to this system must be topological packages.

The relationship of the large $N = 4$ theory to the one with the 'small' $N = 4$ symmetry [3] is very simply stated in terms of these fields. The fields (b, c), (β^a, γ^a) and (b^{+i}, c^{+i}) are the ghost fields of the small algebra and have a total central charge of 12. The remaining eight commuting ghosts and the eight anticommuting ghosts in (3.1) are then interpreted as a 'matter' multiplet of the small $N = 4$ theory with central charge -12. This is the origin of the statement that the critical dimension of this theory is -8. In a certain sense it is a theory in which the rôles of ghosts and matter have been interchanged. [A very different interpretation of the small $N = 4$ theory is given in [20].]

One curiosity is that the large $N = 4$ theory can also be interpreted as a theory with $N = 3$ supersymmetry. This can be seen by integrating out an anomaly-free subset of fields consisting of one of the gravitino ghost pairs (β^4, γ^4), one of the $SU(2)$ ghost packages, (b^{-i}, c^{-i}), three of the internal symmetry fermion ghost pairs, (ϵ^a, δ^a) with $a = 1, 2, 3$, and the $U(1)$ ghost pair, (f, g). These fields contribute respectively 11, -6, -3 and -2 to the central charge. The remaining fields consist of (b, c), (β^i, γ^i), (b^{+i}, c^{+i}), with $i = 1, 2, 3$, and the remaining fermionic ghost system, (ϵ^4, δ^4). This is precisely the content of the theory with $N = 3$ local supersymmetry, which again has vanishing central charge in the gravitational sector. Its invariances include a local $SU(2) \sim O(3)$ gauge symmetry and a local fermionic symmetry of rank one. There is no way to reduce the supersymmetry any further simply by integrating out fields.

However, as with the $(2, 2)$ theory, it can be reduced by twisting fields in a manner that

preserves the vanishing of the total central charge. These deformations are illustrated in table 2.

	$N = 4$			$N = 3$			$N = 2$		
	field	weight	c	field	weight	c	field	weight	c
-	(b, c)	$(2, -1)$	-26	(b, c)	$(2, -1)$	-26	(b, c)	$(2,-1)$	-26
+	(β^1, γ^1)	$(\frac{3}{2}, -\frac{1}{2})$	11	(β^1, γ^1)	$(\frac{3}{2}, -\frac{1}{2})$	11	(β^1, γ^1)	$(\frac{3}{2}, -\frac{1}{2})$	11
+	(β^2, γ^2)	$(\frac{3}{2}, -\frac{1}{2})$	11	(β^2, γ^2)	$(\frac{3}{2}, -\frac{1}{2})$	11	(β^2, γ^2)	$(\frac{3}{2}, -\frac{1}{2})$	11
+	(β^3, γ^3)	$(\frac{3}{2}, -\frac{1}{2})$	11	(β^3, γ^3)	$(\frac{3}{2}, -\frac{1}{2})$	11	$(\beta^3, \gamma^3)^T$	$(\frac{1}{2}, \frac{1}{2})$	-1
+	(β^4, γ^4)	$(\frac{3}{2}, -\frac{1}{2})$	11	$(\beta^4, \gamma^4)^T$	$(\frac{1}{2}, \frac{1}{2})$	-1	$(\beta^4, \gamma^4)^T$	$(\frac{1}{2}, \frac{1}{2})$	-1
-	(b^{-i}, c^{-i})	$(1, 0)$	-6	(b^{-i}, c^{-i})	$(1, 0)$	-6	$(b^{-i}, c^{-i})^T$	$(\frac{1}{2}, \frac{1}{2})$	3
-	(b^{+i}, c^{+i})	$(1, 0)$	-6	$(b^{+i}, c^{+i})^T$	$(\frac{1}{2}, \frac{1}{2})$	3	$(b^{+i}, c^{+i})^T$	$(\frac{1}{2}, \frac{1}{2})$	3
+	(δ^1, ϵ^1)	$(\frac{1}{2}, \frac{1}{2})$	-1	(δ^1, ϵ^1)	$(\frac{1}{2}, \frac{1}{2})$	-1	(δ^1, ϵ^1)	$(\frac{1}{2}, \frac{1}{2})$	-1
+	(δ^2, ϵ^2)	$(\frac{1}{2}, \frac{1}{2})$	-1	(δ^2, ϵ^2)	$(\frac{1}{2}, \frac{1}{2})$	-1	(δ^2, ϵ^2)	$(\frac{1}{2}, \frac{1}{2})$	-1
+	(δ^3, ϵ^3)	$(\frac{1}{2}, \frac{1}{2})$	-1	(δ^3, ϵ^3)	$(\frac{1}{2}, \frac{1}{2})$	-1	(δ^3, ϵ^3)	$(\frac{1}{2}, \frac{1}{2})$	-1
+	(δ^4, ϵ^4)	$(\frac{1}{2}, \frac{1}{2})$	-1	(δ^4, ϵ^4)	$(\frac{1}{2}, \frac{1}{2})$	-1	$(\delta^4, \epsilon^4)^T$	$(1, 0)$	2
-	(f, g)	$(1, 0)$	-2	$(f, g)^T$	$(\frac{1}{2}, \frac{1}{2})$	1	$(f, g)^T$	$(\frac{1}{2}, \frac{1}{2})$	1

TABLE 2: The fields of the holomorphic sector of the $N = 4$ theory and the deformations that reduce the supersymmetry to $N = 3$ and $N = 2$. The purely gravitational $N = 4$ model is taken as the starting point (with $i = 1, 2, 3$). The field assignments in the $N = 2$ theory differ from those in table 1 due to ambiguities in the choice of deformations.

3.1 Deformation to $N = 3$ plus 1 topological package.

Instead of integrating out the anomaly-free subset of fields in passing from the $N = 4$ to the $N = 3$ theory they can be twisted into a set of cancelling weight-$(1/2, 1/2)$ bosonic and fermionic fields. Thus, twisting one pair weight-$(3/2, 1/2)$ gravitino ghost pair, (β^4, γ^4), to weight-$(1/2, 1/2)$ bosonic fields $(\beta^4, \gamma^4)^T$ changes the central charge by -12. This change may be compensated by twisting three of the six weight-$(1, 0)$ fermionic ghosts,(b^{+i}, c^{+i}) associated with one of the internal $SU(2)$ gauge symmetries into three fermionic weight-$(1/2, 1/2)$ fields, $(b^{+i}, c^{+i})^T$. Each of these fields has its contribution to the central charge increased from $c = -2$ to $c = 1$. In addition, the weight-$(1, 0)$ fermionic ghost of the $U(1)$ symmetry, (f, g), can be

transformed into a fermionic weight-$(1/2/, 1/2)$ field, $(f, g)^T$, also changing its central charge by $+3$. The overall increase in the central charge due to the twisting of the $(1, 0)$ ghosts is 12, which cancels the decrease from twisting (β^4, γ^4). These twistings therefore retain the vanishing central charge, $c_{ghost}^{(3)} = 0$, and reduce the supersymmetry, which gives a path to the $N = 3$ theory that is different from that based on integrating out an anomaly-free multiplet of fields.

The field content of this $N = 3$ theory consists of the anomaly-free set of ghost fields – (b, c), (β^i, γ^i), (b^{+i}, c^{+i}), (ϵ^4, δ^4). In addition to this standard content of the $N = 3$ model the remaining fields can be grouped together into topological packages consisting of a pair of opposite statistics $(1/2, 1/2)$ fields such as (ϵ^i, δ^i); $(b^{+i}, c^{+i})^T$ and $(\beta^4, \gamma^4)^T$; $(f, g)^T$. Although there is a great deal of ambiguity in the choice of marginal deformations the above choice is the one which makes the local $SU(2)$ invariance manifest.

3.2 Deforming to $N \leq 2$.

This process can obviously be repeated to obtain a theory with $N = 2$ supersymmetry as illustrated by the transformations in table 2. The field representation of this $N = 2$ theory differs from that used in section 2 but a further set of twists would transform these into the same definitions as in table 1. [A continuous algebraic deformation of the $N = 2$ algebra to the large $N = 4$ algebra appears in [17].]

This demonstrates how a string theory with the large $N = 4$ symmetry (3.1) can be interpreted as an $N = 2$ critical superstring theory, together with a purely topological sigma model based on four fermionic and four bosonic degrees of freedom. String theories with $N = 1$ supersymmetric sectors (the $(2, 1)$ and $(1, 1)$ theories) can now be described by the same deformations of (β^2, γ^2) and the fermionic topological fields that were discussed in section 2. However, only the topological package of fields $(\overline{F}_1^\alpha, F_1^\alpha)$; $(\lambda_1^\alpha, \rho_1^\alpha)$ is obtained in the $N = 2$ theory whereas we saw in section 2 that in order to deform the theory into one with at least one $N = 0$ sector a second set of similar fields is needed.

3.3 Heterotic $N = 4$ theories

Up to this point we have assumed for simplicity that the deformations of the $(4, 4)$ theory are non-chiral so that there are no essentially new issues relating to the constraints between

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left-movers and right-movers that arise in heterotic theories. In considering deformations that take the $(4,4)$ theory to $(4,2)$, $(4,1)$ and $(4,0)$ it is necessary to add extra fields in order to ensure that the fermionic ghosts are non-chiral. This is the analogue of the requirement in the $(2,0)$ theory that the (f,g) pair has both left-moving and right-moving components as shown in table 1. In that case the presence of the extra $c = -2$ right-moving fields led to a total of $c = -28$ from the ghosts that had to be cancelled by the matter. In the $(2,1)$ theory the (f,g) pair were accompanied by $N = 1$ superpartners and the matter sector had $c = 12$.

The $N = 4$ ghost system has a set of fermionic ghosts that contribute $c = -40$ (the $-$ fields in table 2). In deforming to the $(4,2)$ theory we now want to leave the right-moving fermionic ghosts (f,g) and (b^{+i}, c^{+i}) undeformed (*unlike* the deformation illustrated in table 2). In this case it is necessary to append exactly two right-moving topological packages to the original pure $(4,4)$ theory. Then the deformation that twists (β^3, γ^3) and (β^4, γ^4) can be chosen so as to simultaneously twist the fermionic fields in these two packages instead of twisting (f,g) and (b^{+i}, c^{+i}). The resulting ghost fields of the right-moving sector now consist of the original $N = 4$ ghosts, but with two gravitino ghost pairs twisted, giving a total of $c = -24$. It is easy to see that these fields fall into representations of the $N = 2$ supersymmetry. The 'matter' fields come from the two deformed packages which each contribute $c = 12$. These form four $N = 2$ supermultiplets. The resulting theory may therefore be thought of as a $(4,2)$ theory with a target space that has no space-time dimensions but with right-moving fields of the $N = 2$ theory compactified on a self-dual lattice together with extra space and time coordinates that combine with extra ghosts into topological BRST quartets.

This procedure extends to the $(4,1)$ theory if three right-moving topological packages are added to the initial $(4,4)$ theory. The deformations are chosen to twist three of the (β, γ) pairs and the fermionic fields in the packages. The ghosts in the right-moving sector now contribute a total of $c = -36$ which is cancelled by the $c = 36$ 'matter' from the twisted topological packages. This right-moving matter can be arranged in a variety of manners consistent with modular invariance. In particular, this theory has a target space that can be interpreted as that of a zero-dimensional world-volume embedded in a ten-dimensional target space (again there are extra space and time dimensions that combine with ghosts into topological BRST quartets). This is intriguingly suggestive of the 'world-volume' of the D-instanton embedded in the target space of type II superstring theory.

In the $(4, 0)$ theory the right-moving fermionic ghosts contribute $c = -40$. The obvious extension of the above procedure requires a minimum of four right-moving topological packages, $(\overline{F}_r^\alpha, F_r^\alpha); (\lambda_r^\alpha, \rho_r^\alpha)^T$ $(r = 1, \cdots, 4)$, to be added to the pure $N = 4$ theory. Twisting the four gravitino ghost pairs to $(\beta_r^\alpha, \gamma_r^\alpha)^T$ would give $c = -48$. Performing compensating twists of the fermionic fields in the four packages appears to give matter fields with $c = 48$. However, there is some redundancy – the fields $(\lambda_1^\alpha, \rho_1^\alpha)^T$ can be grouped with $(\beta_r, \gamma_r)^T$ in a topological package of $(1/2, 1/2)$ fields with vanishing central charge. Likewise, $(\lambda_2^\alpha, \rho_2^\alpha)^T$ and the fields $(\delta^\alpha, \epsilon^\alpha)$ form a second topological package of $(1/2, 1/2)$ fields. The residual matter has $c = 40$ that cancels the ghosts. Again the target space theory is zero dimensional with the right-moving fields compactified in a manner consistent with modular invariance and giving a net number of 26 zero modes, consistent with an object embedded in 26 dimensions.
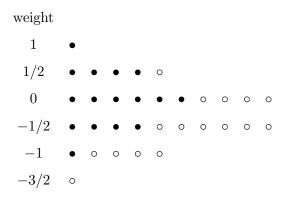
4 Beyond $N = 4$

We have seen that by suitable field redefinitions the fields of critical (N, N') superstring theories with $N, N' \geq 1$ can be reinterpreted as ghost fields of the theory with large $(4, 4)$ supersymmetry. These field redefinitions amount to finite marginal deformations that change the conformal weights of individual fields in a manner that maintains the vanishing of the total central charge. In this way the ghost and matter fields of the sectors with $0 \leq N < 3$ emerge from the $c = 0$ $N = 4$ ghost system. There is no suggestion that these field definitions which embed theories with lower supersymmetry in ones with higher supersymmetry are unique. But, although it is easy to imagine other ways of packaging the fields together, the ones described so far in this paper give a particularly intriguing interpretation in which target-space coordinates emerge from the geometry (the ghost fields) of the large $N = 4$ super world-sheet. Since these target spaces include those of the $(2, 1)$ and $(2, 2)$ theories we are here uniting theories whose target spaces describe space-time with those which describe world-volumes embedded in space-time (and, indeed, those with target spaces that describe the world-volumes of theories whose target spaces are world-volumes, and so on ...).

However, we have seen that it may be necessary to add some topological packages to the purely gravitational sector of the $N = 4$ theory. One possibly unifying observation is that there are supergravities with $N > 4$ which automatically contain such extra fields [13]. In these

theories the ghosts are simply identified as antisymmetric tensor representations of $O(N)$. Their action is an obvious generalization of the gauge-fixed pure world-sheet $N = 4$ theory – a linear ghost system obtained by fixing the gauge in which all the gauge fields of the extended two-dimensional local supergravity vanish. The central charge vanishes for all $N \geq 3$.

Deformations of these theories, analogous to those described in this paper, lead to theories with lower N, but with extra topological packages appended. For example, the following array indicates how the $N = 5$ ghost field content can be described as the sum of the $N = 4$ theory together with an anomaly-free set of eight bosonic and eight fermionic fields,

```
weight
   1      •
  1/2     •  •  •  •  ○
   0      •  •  •  •  •  •  ○  ○  ○  ○
 -1/2     •  •  •  •  ○  ○  ○  ○  ○  ○
  -1      •  ○  ○  ○  ○
 -3/2     ○
```

The ○'s and •'s indicate the $N = 5$ ghosts where the •'s are the ghosts of the $N = 4$ (labelled in the first column). The ○'s indicate the sixteen fermionic and bosonic ghosts that, together with their antighosts, can be deformed by anomaly-free twists into two topological packages of conformal weight $(1, 0)$ fields. Thus, the field content of the $N = 5$ theory is large enough that the non-chiral supersymmetry-reducing marginal deformations described earlier produce the $(2, 2)$ theory depicted in table 1, and hence the theories with $N = 2, 1$ or 0 supersymmetry – all the 'matter' emerging from deformations of the ghost fields. We saw in section 3.3 that extra packages are needed to obtain $(4, N)$ ($N \leq 2$) heterotic theories – these extra fields could be obtained by considering theories with ever-increasing values of N. The structure of the $(4, N)$ theories is particularly intriguing and they surely merit further investigation.

In this paper we have discussed some hints that string world-sheets and their target spaces may be related in an underlying theory. These ideas come from the study of deformations of theories based on two-dimensional world-sheets with extended supersymmetry. Since the 'matter fields' of $N < 3$ theories emerge in this procedure as deformations of the gravitational ghosts of theories with $N \geq 3$, we see that the target space geometry may be encoded in some

larger framework that includes world-sheet structure.

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